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USER MANUAL

BIGRAP

*B*AYESIAN

*I*NTERACTIVE

*G*RAPHICS

*R*ELIABILITY

*A*SSESSMENT

*P*ROCEDURE

By

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Dover, New Jersey

B I G R A P

BAYESIAN

INTERACTIVE

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ABSTRACT

BIGRAP is a package of interactive graphics programs, written for use on the TEKTRONIX 4014 Graphics Terminal connected to the ARRADCOM CDC 6500/6600 computer configuration. This package consists of a set of intricate programs that allows a user to input component success/failure data and a Boolean expression depicting system reliability logic for the purpose of assessing system reliability. The computer converts the logic expression to an algebraic expression for the system reliability as a function of the individual component reliabilities. A Bayesian statistical algorithm is then employed to provide the user with point and confidence interval estimates of system reliability. In addition the graphics feature of the package displays histograms and corresponding Beta distributions involved in the analysis. The programs are run sequentially as a cataloged procedure using the BEGIN/REVERT utility. This report is intended to serve as a guide to potential users of BIGRAP for operation and interpretation of results.

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2. INTRODUCTION AND METHODOLOGY

The Bayesian Interactive Graphics Reliability Assessment Procedure (BIGRAP) is a package of interactive graphics programs written for use on the Tektronix 4014 Graphics Terminal tied into the ARRADCOM CDC 6500/6600 computer system. The package consists of a set of Fortran programs which are executed sequentially as a catalogued procedure using the BEGIN/REVERT utility. Each of the five main programs in BIGRAP, along with its associated subroutines, performs a major computational step required as part of a pseudo-Bayesian approach to reliability assessment of complex systems based on component success/failure data. This report is intended to serve as a BIGRAP user manual, as well as an introductory methodological summary providing the reader with the essentials necessary to use BIGRAP and understand the output it generates.

Reliability assessment approaches for complex systems have historically had two major weaknesses. Firstly, they provided only a point estimate of reliability and failed to provide a measure of the total uncertainty reflected by the limited component data. Secondly, the computational requirements were unwieldy and oriented to individual systems requiring a large amount of time to accomplish with minimal computer aid. BIGRAP has neither of these weaknesses. A Bayesian reliability assessment approach is employed for computing point estimates, as well as confidence limits providing the required measures of uncertainty. The interactive graphics application and BEGIN/REVERT utility make the procedure easy to use, even by users unfamiliar with computer programming.

In this section, the approach underlying BIGRAP will be outlined so that the potential user will have some idea of (1) the preliminary work he will have to do before running BIGRAP to set up his problem; (2) the rationale behind the computations performed by BIGRAP; and (3) the meaning of the output generated by BIGRAP.

The first step in the system reliability assessment approach underlying BIGRAP is the determination of the total number of components and component types which make up the system. Components are considered to be of the same type in this context if they are identical in design and manufacture, i.e. they can be considered, in the statistical sense, to be drawn from the same population. The meaning and importance of the distinction between the total number of components and the number of component types will become clear to the reader as he proceeds.

The next step in the assessment procedure is the construction of a reliability block diagram which depicts the logic by which system success (or failure) occurs as a result of success (or failure) combinations of all the components which comprise the system. While it is likely that most potential users of BIGRAP are familiar with the concept of a reliability block diagram, a simple example will be provided here so that succeeding steps of the assessment approach may be more easily illustrated. Consider the simple system functional schematic provided in Figure 1.

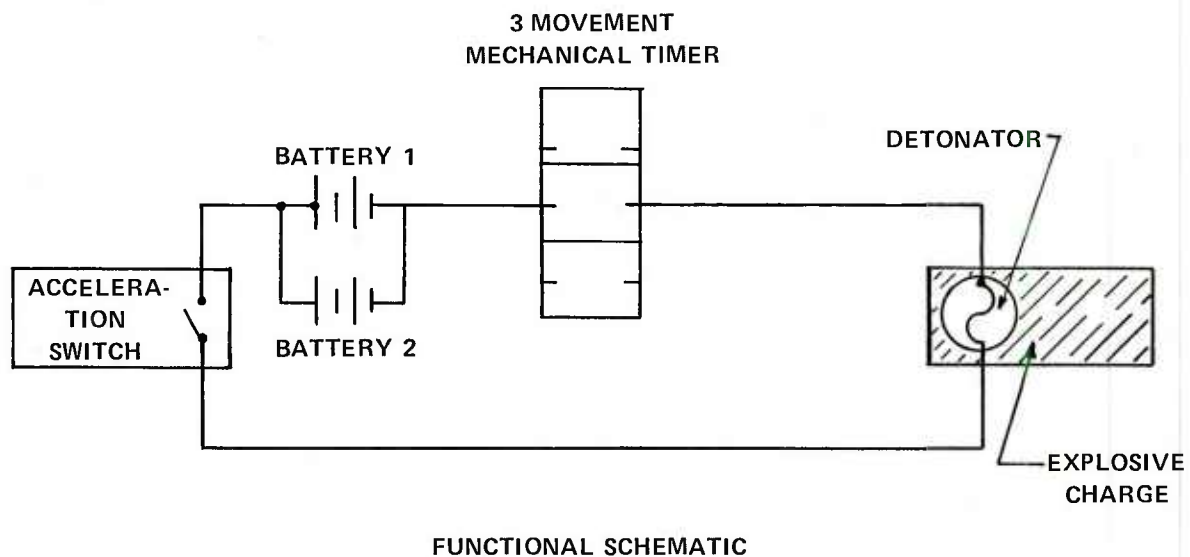
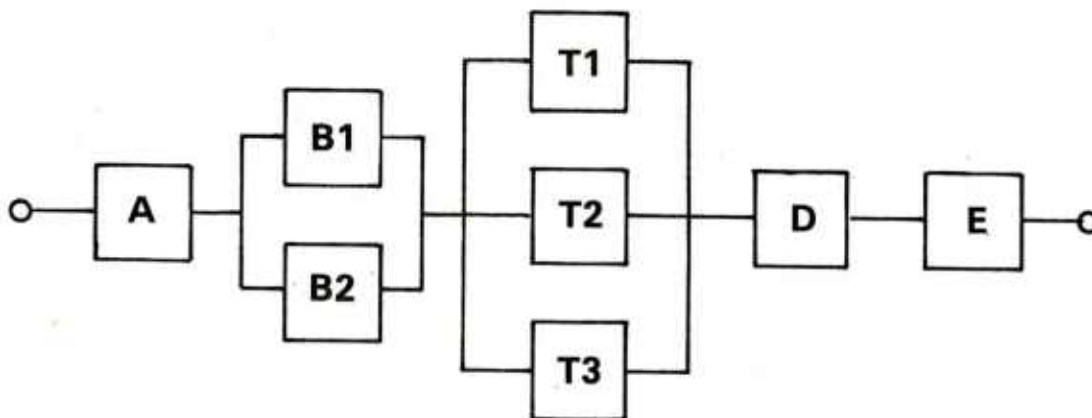


Figure 1



RELIABILITY BLOCK DIAGRAM

Component Name	Component Symbol	Component Type Symbol	Component Type Number
Acceleration Switch	A	A	1
Battery 1	B1	B	2
Battery 2	B2	B	2
Timer Movement 1	T1	T	3
Timer Movement 2	T2	T	3
Timer Movement 3	T3	T	3
Detonator	D	D	4
Explosive Charge	E	E	5

Figure 2

This simple system represents the author's naive conception of a possible (but improbable) missile high explosive warhead section. The system is designed so that upon sensing sufficient acceleration after launch the acceleration switch closes and the three identical movements of the timer are started. When a preset time is measured by any one of the three timers the circuit is closed between the detonator and the two identical batteries, each having sufficient voltage to fire the detonator. The firing of the detonator ignites the explosive charge completing the system mission. The system has a total of eight components, but only five component types. Figure 2 lists the different components along with their assigned component symbols and type symbols below the reliability logic block diagram for the system. The block diagram reflects that components A, D, E, one or both of components B1 and B2, and at least one of components T1, T2 and T3 must function successfully to achieve system success.

BIGRAP was developed with the intention of allowing the user to provide, as part of the input, the system logic as depicted by the reliability block diagram. Boolean algebra provides the tools with which this can be accomplished. Once the reliability block diagram has been constructed it is easy to formulate the equivalent Boolean algebraic expression. For the sample system of Figure 1 the Boolean expression for system success, S, is

$$S=A*(B1 + B2)*((T1 + T2) + T3)*D*E$$

where the symbols "*" denotes "and" logic and "+" denotes "or" logic. If, in this expression, one assigns to the component symbols 1 for success or 0 for failure and then evaluates the expression according to the following simple rules:

$$\begin{array}{ll} 0*0 = 0 & \\ 1*1 = 1 & \\ 0*1 = 1*0 = 0 & \text{(Note: The usual associative,} \\ 1+1 = 1 & \text{distributive and commutative} \\ 0+1 = 1+0 = 1 & \text{axioms hold as well)} \\ 0+0 = 0, & \end{array}$$

a value of S=1 or S=0 will result in implying that system success or failure, respectively, is achieved by the assigned component successes and/or failures. For example, if one assigns A=1, B1=0, B2=1, T1=1, T2=1, T3=0, D=1, and E=1 the Boolean expression with these values substituted becomes

$$S = 1*(0+1)*((1+1)+0)*1*1$$

which can be reduced via the above rules to

$$S = 1*1*(1+0)*1*1$$

and further to

$$S = 1*1*1*1*1 = 1$$

representing system success. As another example consider the assignment $A=1$, $B1=0$, $B2=0$, $T1=1$, $T2=0$, $T3=0$, $D=1$, $E=1$ resulting in

$$\begin{aligned} S &= 1*(0+0)*((1+0)+0)*1*1 \\ &= 1*0*(1+0)*1*1 \\ &= 1*0*1*1*1 \\ &= 0 \end{aligned}$$

showing that this latter assignment results in system failure. For readers for which the above explanation does not sufficiently clarify the process of translating the block diagram to a Boolean expression, many papers and texts in the extant literature provide more detailed explanations (see references 11 and 12).

Boolean algebra provides the means by which the system event can be described in terms of component events. The subject of reliability, however, deals with probabilities, and in particular the probability of the system success event. In fact the system reliability is defined here as the probability of system success. System reliability assessment based on component data involves the computation of point and confidence interval estimates of system reliability using success/failure data which directly estimates the individual component reliabilities. The component reliabilities are defined analogously as the probabilities of component success. Components of the same type have, by definition, the same reliability value. To proceed with the reliability assessment approach, therefore, a procedure is required by which the Boolean logic expression can be converted to an algebraic expression for system reliability in terms of the component type reliabilities. The first three programs executed as part of BIGRAP perform such a conversion for a Boolean expression of a system provided as input. The method employed therein to accomplish this is too lengthy to describe here but is adequately explained in References 11 and 12. The user need only be acquainted with the form of the results which will be illustrated herein.

The conversion is accomplished in two stages. The first stage involves determination of the total number of success paths (or tie sets). For our example there are six such success paths. They are:

A*B1*T1*D*E
A*B1*T2*D*E
A*B1*T3*D*E
A*B2*T1*D*E
A*B2*T2*D*E
A*B2*T3*D*E

The second stage uses this essentially expanded form of the Boolean expression and converts it to an ordinary algebraic expression for the system reliability, R_s ; in terms of the component type reliabilities, resulting in

$$R_s = 6R_A R_B R_T R_D R_E - 6R_A R_B R_T^2 R_D R_E \\
+ 2R_A R_B R_T^3 R_D R_E - 3R_A R_B^2 R_T R_D R_E \\
+ 3R_A R_B^2 R_T^2 R_D R_E - R_A R_B^2 R_T^3 R_D R_E$$

where an R_x on the right side of the equal sign denotes the reliability of component type X. This expression provides the required relationship between the system reliability and the component type reliabilities.

The Bayesian aspects of the assessment procedure are introduced at this point. The basic underlying assumption is that the unknown component type reliabilities (measured by the success/failure data) can be considered as independent random variables in the Bayesian sense, each having a prior probability distribution belonging to the family of beta distributions. The general form of the beta probability density function (pdf) for a random variable, R , is:

$$f_{\beta}(R|A,B) = \frac{\Gamma(A+B)}{\Gamma(A)\Gamma(B)} R^{A-1}(1-R)^{B-1} \text{ for } 0 \leq R \leq 1 \\
= 0 \quad \text{otherwise.}$$

where: (i) A and B are the parameters of the beta distribution with the restriction that $A > 0$ and $B > 0$.

(ii) $\Gamma(\cdot)$ is the complete gamma function defined by

$$\Gamma(x) = \int_0^{\infty} y^{x-1} e^{-y} dy$$

having the property that $\Gamma(x+1) = x\Gamma(x)$ so that for a positive integer, n ,

$$\Gamma(n+1) = n!$$

If the component types are indexed 1 thru m , where m is the number of types ($m = 5$ for the example) the prior beta pdf of each component type reliability will be represented by $f_{\beta}(R_i | A_i, B_i)$, $i=1, \dots, m$. BIGRAP allows the user to select from a variety of options for defining the A_i and B_i values. These options will be described in Section 5b of this report.

The data is introduced into the assessment procedure by similarly indexing the component type number of successes and number of failures as s_i and f_i respectively. For a given component type reliabilities, R_i , $i=1, \dots, m$, the likelihood of observing the given data s_i, f_i is then provided by the binomial distribution with probability mass function given by

$$\Pr\{s_i, f_i; R_i\} = \frac{(s_i + f_i)!}{s_i! f_i!} R_i^{s_i} (1-R_i)^{f_i}$$

The likelihood expressed as a function of R_i is then

$$L(R_i; s_i, f_i) = \frac{(s_i + f_i)!}{s_i! f_i!} R_i^{s_i} (1-R_i)^{f_i}$$

Application of Bayes Theorem (Principle of Inverse Probability), which says that the posterior (updated with data) probability is proportional to the product of the prior probability and the likelihood, results in the following:

$$\text{Posterior pdf of } R_i = \frac{f_{\beta}(R_i | A_i, B_i) L(R_i; s_i, f_i)}{\int_0^1 f_{\beta}(x | A_i, B_i) L(x; s_i, f_i) dx}$$

The expression on the right can, upon substitution of the component type prior beta pdf and binomial likelihood, be reduced to

$$f_{\beta}(R_i | A_i + s_i, B_i + f_i)$$

so that the posterior distribution of R_i is also a member of the beta family with posterior parameters given by

$$AX_i = A_i + s_i$$

$$BX_i = B_i + f_i$$

It is, therefore, a simple matter to add the component success/failure data into the assessment procedure for a given set of component type prior distributions.

Since each of the component type reliabilities is considered a random variable, the system reliability, R_s , which is a function of the component type reliabilities, is also a random variable. The exact form of the probability distribution of R_s clearly depends on the distribution of the component type reliabilities and the relationship of the component type reliabilities to system reliability as reflected in the algebraic reliability expression for R_s . In-depth investigations have shown that this exact distribution can be reasonably approximated by a member of the beta family.

BIGRAP uses two methods for determining the appropriate beta fit to the distribution of R_s . Monte Carlo sampling and the method of moments. The Monte Carlo approach involves generation of a large number of "trials", where on each trial each of the component type reliability posterior distributions $f_\beta(R_i | AX_i, BX_i)$, $i = 1, \dots, m$, is randomly sampled yielding a set of type reliability values R_1, R_2, \dots, R_m . These values are substituted into the algebraic reliability expression and a value of R_s , the system reliability, is obtained. This process is repeated for the total number of trials specified by the user. A histogram depicting the distribution of the resulting system reliability values is constructed. In addition the arithmetic mean, \bar{R}_s , and sample variance, s_R^2 , of the R_s values is computed. A direct relationship between the parameters of a beta distribution and its mean and variance is then used, with R_s substituted for the mean and s_R^2 substituted for the variance, to estimate the parameters of a system beta distribution serving as a beta fit to the histogram.

In discussing the method of moments a few basics should be recognized concerning the moments of continuous random variables in general and beta distributed random variables in

particular. It is assumed that the reader is familiar with the concept of expectation. The r th moment of a random variable, X , is defined to be the expectation of the r th power of X , i.e.

$$r^{\text{th}} \text{ moment of } X = E(X^r)$$

The first moment is referred to as the mean, μ , of the distribution of the random variable. Its variance, σ^2 , is defined as the second moment about the mean. Therefore,

$$\mu = E(X)$$

$$\sigma^2 = E(X - \mu)^2$$

or equivalently

$$\sigma^2 = E(X^2) - E(X)^2$$

This latter form of the expression for σ^2 follows from the following. If X and Y are random variables (or functions of random variables) and C_1 and C_2 , constants, then

$$E(C_1 X + C_2 Y) = C_1 E(X) + C_2 E(Y)$$

If X and Y are independent, then

$$E(XY) = E(X)E(Y)$$

For a random variable R having a beta distribution with pdf given by $F_\beta(R|A,B)$ the r^{th} moment is

$$E(R^r) = \prod_{i=1}^r \frac{A+i-1}{A+B+i-1}$$

This result then yields the following expression for μ and σ^2 of the beta distribution

$$\mu = A/(A+B)$$

$$\sigma^2 = \frac{A}{A+B} \left(\frac{A+1}{A+B+1} - \frac{A}{A+B} \right)$$

By solving these two expressions for A and B in terms of μ and σ^2 the following useful formulae are obtained:

$$A = \frac{\mu^2(1-\mu)}{\sigma^2} - \mu$$

$$B = \frac{\mu(1-\mu)^2}{\sigma^2} + \mu - 1$$

By substituting estimates of μ and σ^2 into these last two expressions, estimates of A and B can be obtained. This is precisely what was done in determining the beta fit to the Monte Carlo histogram wherein \bar{R}_S and s_R^2 were used as estimates of μ and σ^2 , respectively.

The method of moments involves the computation of the first and second moments of the posterior distribution of system reliability in terms of the moments of the component type reliability posterior distributions. The algebraic reliability expression for R_S and its algebraic square for R_S^2 are used for this purpose. The example system, with the type numbers listed in Figure 2, can be used to illustrate this procedure. The first moment of R_S is first determined as:

$$E(R_S) = E(6R_1R_2R_3R_4R_5 - 6R_1R_2R_3^2R_4R_5 \\ + 2R_1R_2R_3^3R_4R_5 - \text{etc.})$$

Using what has been stated about expectations, this can then be expressed as

$$E(R_S) = 6E(R_1)E(R_2)E(R_3)E(R_4)E(R_5) \\ - 6E(R_1)E(R_2)E(R_3^2)E(R_4)E(R_5) \\ + 2E(R_1)E(R_2)E(R_3^3)E(R_4)E(R_5) \\ - \text{etc.}$$

By substituting the appropriate expression for each beta moment in terms of the component type posterior beta

parameters, $E(R_s)$ can be evaluated. The appropriate substitutions follow from:

$$E(R_i^r) = \prod_{j=1}^r \frac{AX_i + j - 1}{AX_i + BX_i + j - 1}$$

$$\text{e.g. } E(R_1) = \frac{AX_1}{AX_1 + BX_1}, \quad E(R_3^2) = \frac{AX_3}{AX_3 + BX_3} \cdot \frac{AX_3 + 1}{AX_3 + BX_3 + 1}$$

$$E(R_3^3) = \frac{AX_3}{AX_3 + BX_3} \cdot \frac{AX_3 + 1}{AX_3 + BX_3 + 1} \cdot \frac{AX_3 + 2}{AX_3 + BX_3 + 2}$$

Similarly $E(R_s^2)$ is evaluated using the expectations of the algebraic expression for the square of R_s , i.e.,

$$\begin{aligned} E(R_s^2) = & E(36R_1^2 R_2^2 R_3^2 R_4^2 R_5^2 - 72R_1^2 R_2^2 R_3^3 R_4^2 R_5^2 \\ & + 24R_1^2 R_2^2 R_3^4 R_4^2 R_5^2 - 36R_1^2 R_2^3 R_3^2 R_4^2 R_5^2 \\ & + \text{etc.}) \end{aligned}$$

By setting $\mu_s = E(R_s)$ and $\sigma_s^2 = E(R_s^2) - E(R_s)^2$

in the expressions for the beta A and B parameters, the method of moments posterior beta distribution for system reliability, R_s , is fully determined by the resultant A_s and B_s values; i.e.,

$$\begin{aligned} A_s &= \frac{\mu_s^2 (1 - \mu_s)}{\sigma_s^2} - \mu_s \\ B_s &= \frac{\mu_s (1 - \mu_s)^2}{\sigma_s^2} + \mu_s - 1 \end{aligned}$$

The result obtained by either the Monte Carlo method or the method of moments is a beta distribution which approximates very closely the true posterior distribution of the system reliability, R_s . In each case the resultant beta distribution

is completely defined by its two parameters, A and B, which will be denoted by A_s and B_s since they are the parameters of the posterior distribution of R_s . The user, however, is generally performing the assessment procedure for the purpose of obtaining point or confidence interval estimates of the system reliability. BIGRAP extracts these estimates from the information contained in the system posterior beta distribution. In all cases, the mean and mode (if it exists and is between 0 and 1.0) of the posterior distribution of R_s are provided. These values are computed as follows:

$$\text{Mean of } R_s = A_s / (A_s + B_s)$$

If $A_s > 1$ and $B_s > 1$, then

$$\text{Mode of } R_s = (A_s - 1) / (A_s + B_s - 2)$$

The recommended strategy for point estimation is to take as the point estimate of R_s the mode when A_s and B_s are both greater than (not equal to) 1.0 and the mean, otherwise.

For the purpose of confidence interval estimation, BIGRAP presently provides the lower 50, 90, and 95 per cent Bayesian confidence limits. The lower 100 $(1-\gamma)\%$ confidence limit for $\gamma = .5, .1$, or $.05$ is defined as the value of R satisfying the following:

$$\gamma = \int_0^R f(R_s | A_s, B_s) dR_s$$

where the A_s and B_s that are used result from either the Monte Carlo method or the method of moments. Precise numerical integration routines included in BIGRAP set up and interpolate a table of γ vs R to determine the appropriate value of R.

For a particular system configuration to which BIGRAP is being applied for the first time the user may wish to employ both the Monte Carlo approach and the method of moments. By comparing the point and confidence limit estimates resulting from the both methods the user can satisfy himself that the assumptions involved in the method of moments approach (use of only first two moments and a beta fit to system reliability distribution) do not introduce significant errors. In subsequent applications to the same system configuration the user need only apply the method of moments should the comparison be favorable. If, on the other hand, there is significant disagreement between these results, or if the beta distribution in the Monte Carlo approach does not appear to fit the histogram, the user may then choose to continue with the Monte Carlo approach. In the latter case the user should make use of the histogram itself, rather than its beta fit, to extract empirical point and interval estimates.

The succeeding sections of this report present a detailed description of the operation and structure of the BIGRAP software. Instructions for job control and input are also provided. Several examples of the use of BIGRAP are presented. In addition the complex manual computations, which are the alternative to employing BIGRAP, for a simple system configuration are exhibited to emphasize the value of the BIGRAP package.

The reader interested in more details regarding the theoretical basis for BIGRAP should look to the cited references and in particular Reference 14.

As a final introductory comment, the authors wish to point out that a large amount of work was performed in comparing the results from BIGRAP to other approaches to reliability assessment both classical and Bayesian. For the restricted cases (in the sense of system configuration complexity) to which these other approaches could be applied, BIGRAP performed at least as well and, in most cases, better. So whether the reader considers himself (or herself) a Bayesian, Classicist, or "middle-of-the-roader" the assessment algorithm of BIGRAP should prove satisfactory.

3. DESCRIPTION OF THE MAIN PROGRAMS

The Bayesian Interactive Graphics Reliability Assessment Procedure is made up of five main computer programs (BOLN1, BOLN2, BOLN3, MOMENT, MONTI) which will be described here:

Program BOLN1 - Uses as input, the Boolean expression of a reliability block diagram in free form. The program transforms the simple expression to integer form by comparing each element in the expression to the list of special characters (plus sign, asterisk, and right and left parentheses) and components and places the number of the list that equals the element of the expression in the same position as the element. The program then goes through the expression and stores the number of sections and the locations of the plus signs and left and right parentheses for each set of parentheses. The results of the above operations are written on a disk for use in program BOLN2.

Program BOLN2 - Reads the disk output from program BOLN1 as input. The program expands the expression in the conventional manner to ascertain all the success paths. As each path is derived, it is compared to all previously generated paths. If the newly derived path contains or is equal to any previously generated path, the new one is eliminated as a redundancy. If the newly derived path is contained in any other previously generated path, the new one replaces the older one, which is then eliminated as a redundancy. If neither of the above is found to be true, the newly ascertained path is added to the list of non-redundant paths. The results are then written (on both disc and printout).

Program BOLN3 - Converts Boolean logic expression to an expanded algebraic reliability expression. Constructs a Fortran subroutine, on TAPE 12, for calculation of system reliability using the algebraic expression, passes this subroutine to program MONTI, passes the algebraic reliability expression to output and program MOMENT on TAPE 7.

Program MOMENT - Allows the user to select one of four options. Option 1 uses the algebraic reliability expression from program BOLN3. It then determines and stores the square of the algebraic expression and formulates the system distribution 1st and 2nd moments as functions of the moments of the component distributions. These results are then used to determine the equal component prior beta parameters A' and B' , which yield a uniform prior distribution for the system reliability. It also computes the posterior beta parameters for each component and substitutes these results into the expressions for the system moments. The method of moments estimates of system posterior beta parameters are then computed.

In Option 2 and Option 4 the user provides the prior beta parameters (either equal or unequal) for each component. There is, of course, no need for a solution to the moment equations prior to performing the method of moments estimation procedure.

In Option 3 the moment expressions are set equal to user specified 1st and 2nd prior moments. These equations are then solved and the same procedures are followed as in Option 1.

Program MONTI - Uses the component posterior beta parameters for each component as well as the method of moments estimates of the system beta parameters as input. The component parameters are used to define beta distributions for each component. Monte Carlo simulation is then performed by sampling in turn each of the component distributions for a component reliability value; substituting these values into the system algebraic reliability expression (generated in BOLN3), generating a system reliability value on each Monte Carlo trial. After a sufficiently large number of trials, a histogram of system reliability values is obtained, in addition to the mean and variance of the generated system reliability values. It then computes lower confidence limits for system reliability based on the method of moments and Monte Carlo results. Finally the histogram is displayed on the Tektronix screen with a beta curve fit from the mean and variance superimposed. A useful feature of program MONTI is the capability of generating additional histograms for the same set of inputs varying either the number of histogram cells (increments) or the number of Monte Carlo trials (samples). The random number generator subroutines, RDMIN and RDMOUT, are also employed to avoid the use of the same or overlapping set of random numbers on subsequent runs with the same set of inputs. These features are illustrated in section 5 of this report.

The present form of Programs BOLN1, BOLN2, and BOLN3 were based on programs originally written by Howard Corneilson. Each of his original separate programs were run as batch jobs. The modified programs are used interactively with the graphics terminal and are run sequentially using BEGIN/REVERT.

Program MOMENT was based on a batch program written by Bruce D. Barnett. The BIGRAP form of the program derives its input directly from program BOLN3 without the need for the user to provide the algebraic expression.

Program MONTI was based on two programs written by Irving E. Teviovitz for the 274/1700/6500 large scale refresh graphics facility. One program illustrated a user specified beta distribution probability density function and the second program illustrated a beta random distributed histogram. The two programs were combined and converted for use on the Tektronix terminal. The JBETA subroutine in MONTI for numerical integration of the beta distributions was developed by Joseph E. Bay through modification of subroutines used in program RELY (Neff 1969).

The BEGIN/REVERT utility, which automatically executes all the job control statements that are needed to run these programs, is a set of programs developed by the University of Washington.

4. TERMINAL INITIALIZATION AND LOGIN

A typical Tektronix 4014 Graphics Terminal set-up is pictured in Figure 3. A complete description of the terminal and its operation can be obtained in Reference 3.

The purpose of this section is to acquaint the potential user with some basic information required to operate the 4014 terminal in connection with use of BIGRAP.

Figure 4 provides a description of the keyboard configuration of the 4014 terminal.



Figure 3
Typical Tektronix 4014
Graphics Terminal

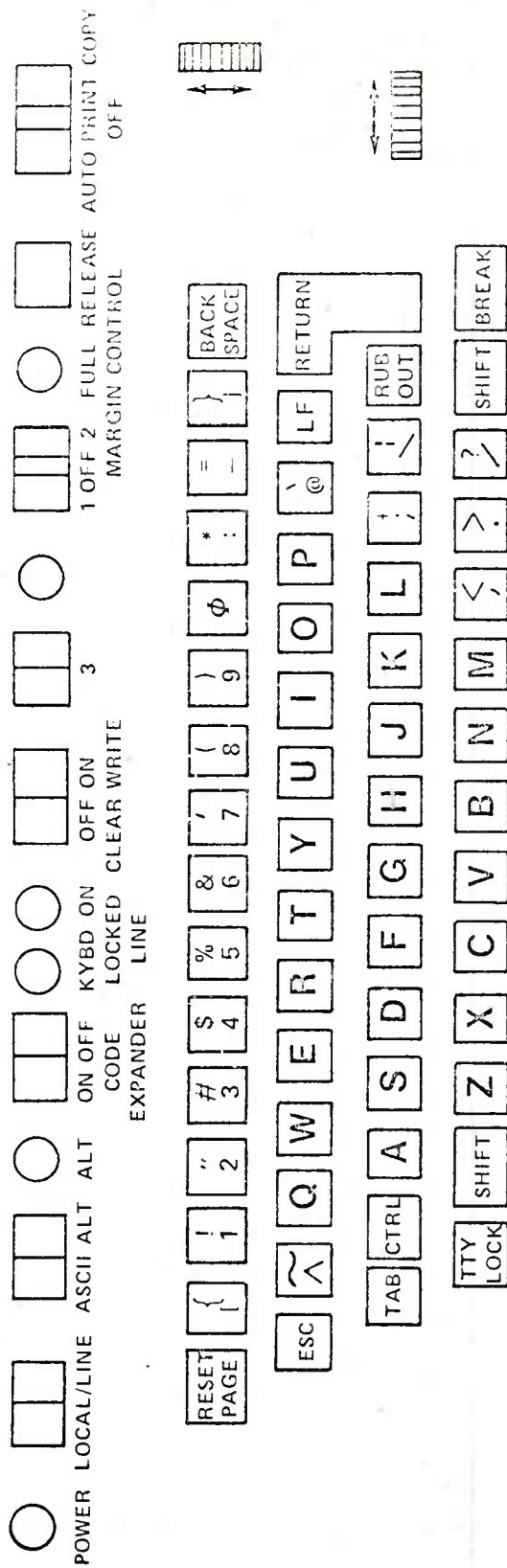


Figure 4

This is the Keyboard of the
4015 Computer Graphics Terminal

The following procedures are required for Terminal initialization.

1. Place the LOCAL/LINE switch to LOCAL (the terminal will be isolated from the computer and causes keyboard inputs to be executed by the terminal).
2. Turn the Terminal ON (the power switch is on the front lower right of the console (one foot from floor). The green power light on the left of the keyboard will light and the display screen will become bright after a few seconds.

NOTE: At turn on, the terminal is in Alpha Mode with the cursor at HOME position with normal (large character size and is in synchronous communications mode.

3. Turn on hard copier (if one is available). Erase one display by pushing the RESET-PAGE key (at least one erase after power-up is required to start the interface).

4. Set the following keyboard switches:

CODE EXPANDER	TO	OFF
CLEAR WRITE	TO	OFF

5. Set the interface control switches

Rotary baud switch to 300 for 300 baud rate (rear of terminal) ASCII/BCD switch to ASCII (rear of terminal)

6. SHIFT, RESET-PAGE keys are depressed on the keyboard simultaneously. This will initialize the Terminal, by placing initial conditions on the Terminal circuits.
7. Press CTRL, SHIFT and P keys - (for local echo).
8. Press the following three keys sequentially:
RETURN ESC : (For small characters)
9. Put LOCAL/LINE switch to LINE
10. Remove grey phone from the modern cradle, switch phone to TALK. Phone select switch under table to be in clockwise position marked "OPT".
11. Dial up (to connect to CDC 6500/6600)
12. When the computer gives an audible tone, the button marked DATA is pushed and the phone is hung up.

13. INTERCOM sign-on (LOGIN) procedures will then be requested on the terminal screen and be responded to by the user as illustrated in figure 5.

5. ILLUSTRATIVE EXAMPLE

INTERCOM commands to initiate the BIGRAP procedure also illustrated in the same figure.

To show what happens next, the example from the Introduction (Section 2) will be used.

The screen is cleared and information shown on page 27 is requested. The user's inputs are underlined.

The name, type, number of successes and failures of each component is entered one at a time, only when prompted and each followed by a return.

Note the warning for entering the Boolean expression.

At the end of the page a Copy may be made and/or the "ESC" key followed by the "Return" key must be depressed (this sequence must be followed at the end of every page).

Systems messages are printed out on page 30.

The output from BOLN 2 and BOLN 3 are batched to terminal specified above.

This output consists of system success paths shown on page 28 and algebraic reliability expression, page 29.

Various options are requested on page 31.

A "Copy,Esc,Return-(C,E,R)" sequence is required here. Various results are shown on page 32.

A "C,E,R" sequence is used at end of "THE VAR OF RS= ".

On page 33 RANDOM START is any ten digit integer number. The NUMBER of Increments and Samples are floating point numbers. A "C,E,R," sequence is used at end of "RANDOM END IS ".

The histogram and distribution curve is shown on page 34. A "C,E,R" is required at the end of cur e. Next follow directions on page 34.

Pages 35 to 38 show how the histogram changes as the number of increments change (line 2 of page 35 and 37.)

The display on page 35 is shown when the cursor is placed on line 3 and the number "3" is pressed.

Pages 39 to 50 show how Options 2,3 and 4 are used.

CONTROL DATA INTERCOM 4.5
DATE 06/08/78
TIME 09.56.41.

PLEASE LOGIN
LOGIN

ENTER USER NAME-UUUUXXXXYY

■■■■■■■■ ENTER PASSWORD-

06/08/78 LOGGED IN AT 09.57.16.
 WITH USER-ID PG
 EQUIP/PORT 40/006
COMMAND- ETL,300 (Extend Time Limit)

COMMAND- BEGIN,RELY,BAYE/MARDO,XX,YYY

(where XX is your batch terminal Id and YYY is 65I or 66B)

Figure 5
LOGIN Procedures (User Inputs are Underlined)

RESULTS FROM BOLN2%

THESE ARE A SUCCESS DATA FOR SYS1

A	* R2	* T3	* 0	* F
A	* R2	* T2	* 0	* F
A	* R2	* T1	* 0	* F
A	* B1	* T3	* 0	* F
A	* R1	* T2	* 0	* F
A	* R1	* T1	* 0	* F

RESULTS FROM ROLN3

THERE ARE A TOTAL OF 5 DIFFERENT TYPES OF COMPONENTS FOR EQUATION SYS1
THEY ARE LISTED BELOW

1. A	2. B	3. T	4. D	5. E
------	------	------	------	------

THERE ARE A TOTAL OF 6 ALGEBRAIC EQUATION SETS FOR THIS EQUATION.
THEY ARE LISTED BELOW

$$+(\quad 6 \quad) * (A \quad) * (R \quad) * (T \quad) * (D \quad) * (E \quad)$$

$$+(\quad -6 \quad) * (A \quad) * (R \quad) * (T \quad) * (D \quad) * (E \quad)$$

$$+(\quad 2 \quad) * (A \quad) * (R \quad) * (T \quad) * (D \quad) * (E \quad)$$

$$+(\quad -3 \quad) * (A \quad) * (R \quad) * (T \quad) * (D \quad) * (E \quad)$$

$$+(\quad 3 \quad) * (A \quad) * (R \quad) * (T \quad) * (D \quad) * (E \quad)$$

$$+(\quad -1 \quad) * (A \quad) * (R \quad) * (T \quad) * (D \quad) * (E \quad)$$

STOP PROGRAM - BOOLEAN ONE (BOLN1)
HAS JUST ENDED
.700 CP SECONDS EXECUTION TIME
STOP 2
.116 CP SECONDS EXECUTION TIME
STOP PROGRAM - BOOLEAN THREE (BOLN3)
HAS JUST ENDED
.926 CP SECONDS EXECUTION TIME

OPTIONS AVAILABLE

- 1 UNIFORM SYSTEM PRIOR
 - 2 USER SPECIFIED EQUAL PRIOR PARAMETERS
 - 3 USER SPECIFIED FIRST AND SECOND SYSTEM MOMENTS
 - 4 USER SPECIFIED UNEQUAL PRIOR PARAMETERS
- OPTION DESIRED = 1**

STOP 4
 1.723 CP SECONDS EXECUTION TIME
 .947 CP SECONDS COMPILATION TI
 ME
 PFN IS
 TEK30
 EOI ENCOUNTERED AFTER COPY OF FILE
 0, RECORD 17

THE PRIOR BETA PARAMETERS FOLLOW
 THE A IS EQUAL TO 1.29243
 THE B IS EQUAL TO .26113

RESULTS USING THE METHOD OF MOMENTS
 THE AS IS EQUAL TO 63.35904
 THE BS IS EQUAL TO 12.10509
 THE MEAN OF RS = .83959
 THE MODE OF RS = .84884
 THE VAR OF RS = .00176

RANDOM START IS 1426673931

THE NUMBER OF INCREMENTS IS 50.0
NUMBER OF SAMPLES IS 2000.0

CONFIDENCE LIMITS FOR METHOD OF MOMENTS FOLLOW

AS = 63.35904 BS = 12.10509

THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .84258
THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .78382
THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .76564

CONFIDENCE LIMITS FOR MONTE CARLO METHOD FOLLOW

AS = 60.57172 BS = 11.05277
MEAN RS = .83866 VAR RS = .00185
MODE RS = .84830

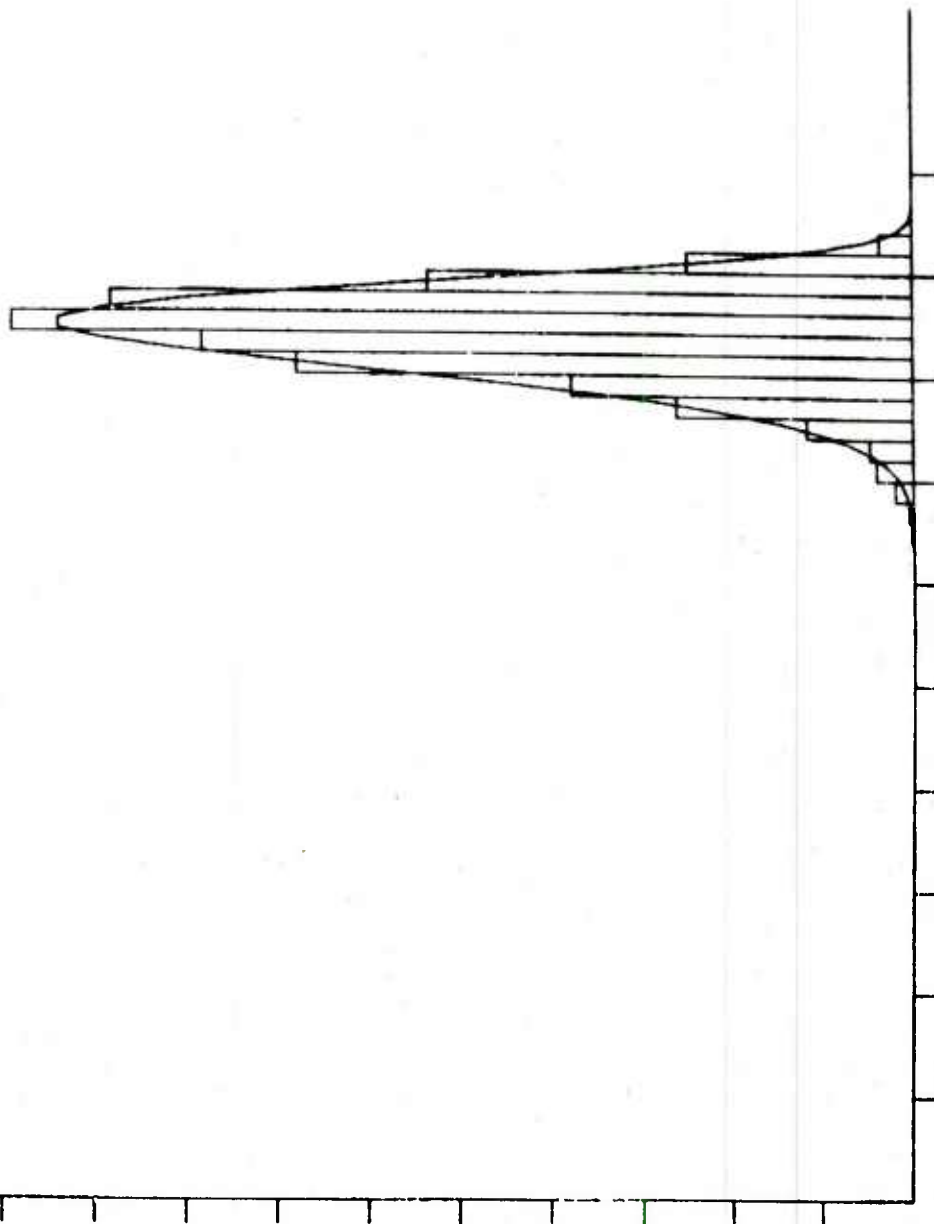
THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .84178
THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .78155
THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .76274

RANDOM END IS 818394315

PLACE CURSOR ON THIS LINE AND PRESS PROPER KEY FOR DESIRED ACTION

KEY ACTION

- 1 JOB TERMINATION
- 2 NEXT CASE-NEW A AND B
- 3 NEXT CASE-OLD A AND B



a. Changing Number of Increments
 This display is shown when the cursor is
 placed on line 3 and the number "3" is pressed.

```

RANDOM START IS 1425364758
THE NUMBER OF INCREMENTS IS 100.00
NUMBER OF SAMPLES IS 2000.0

CONFIDENCE LIMITS FOR METHOD OF MOMENTS FOLLOW
AS = 63.35904 RS = 12.10509

    THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .84258
    THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .78382
    THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .76564

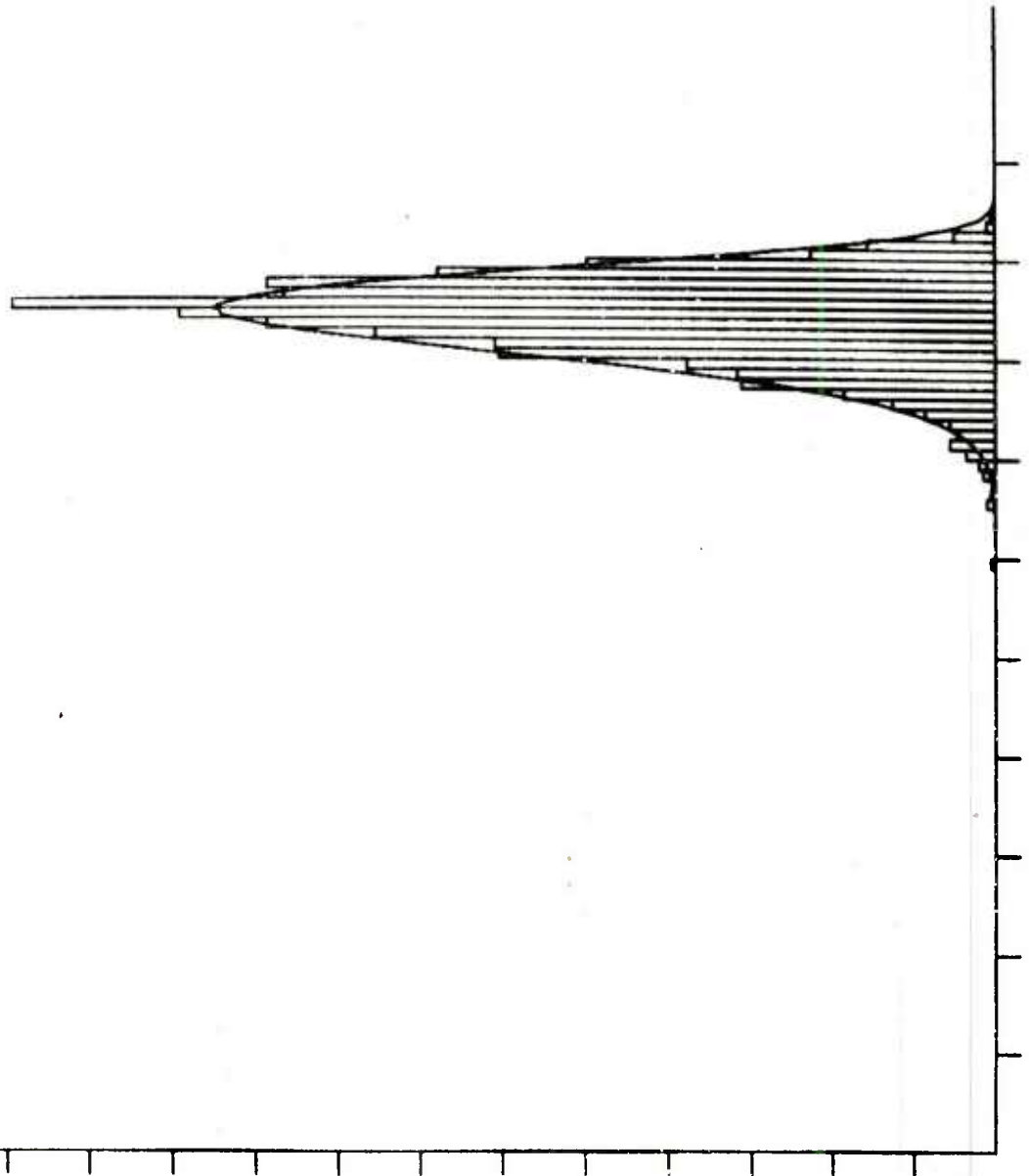
CONFIDENCE LIMITS FOR MONTE CARLO METHOD FOLLOW
AS = 61.06368 RS = 11.67882
MEAN RS = .82854 UMR RS = .00183
MODE RS = .84914

    THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .84265
    THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .78271
    THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .76406

RANDOM END IS 910057110
  
```

PLACE CURSOR ON THIS LINE AND PRESS PROPER KEY FOR DESIRED ACTION

KEY	ACTION
1	JOB TERMINATION
2	NEXT CASE-NEW A AND B
3	NEXT CASE-OLD A AND B



RANDOM START IS 1234567890

THE NUMBER OF INCREMENTS IS 150.0
NUMBER OF SAMPLES IS 2000.0

CONFIDENCE LIMITS FOR METHOD OF MOMENTS FOLLOW

AS = 63.35904 BS = 12.10509

THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .84258
THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .78382
THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .76564

CONFIDENCE LIMITS FOR MONTE CARLO METHOD FOLLOW

AS = 62.90829 BS = 12.15581
MEAN PS = .83806 VAR PS = .00178
MODE PS = .84731

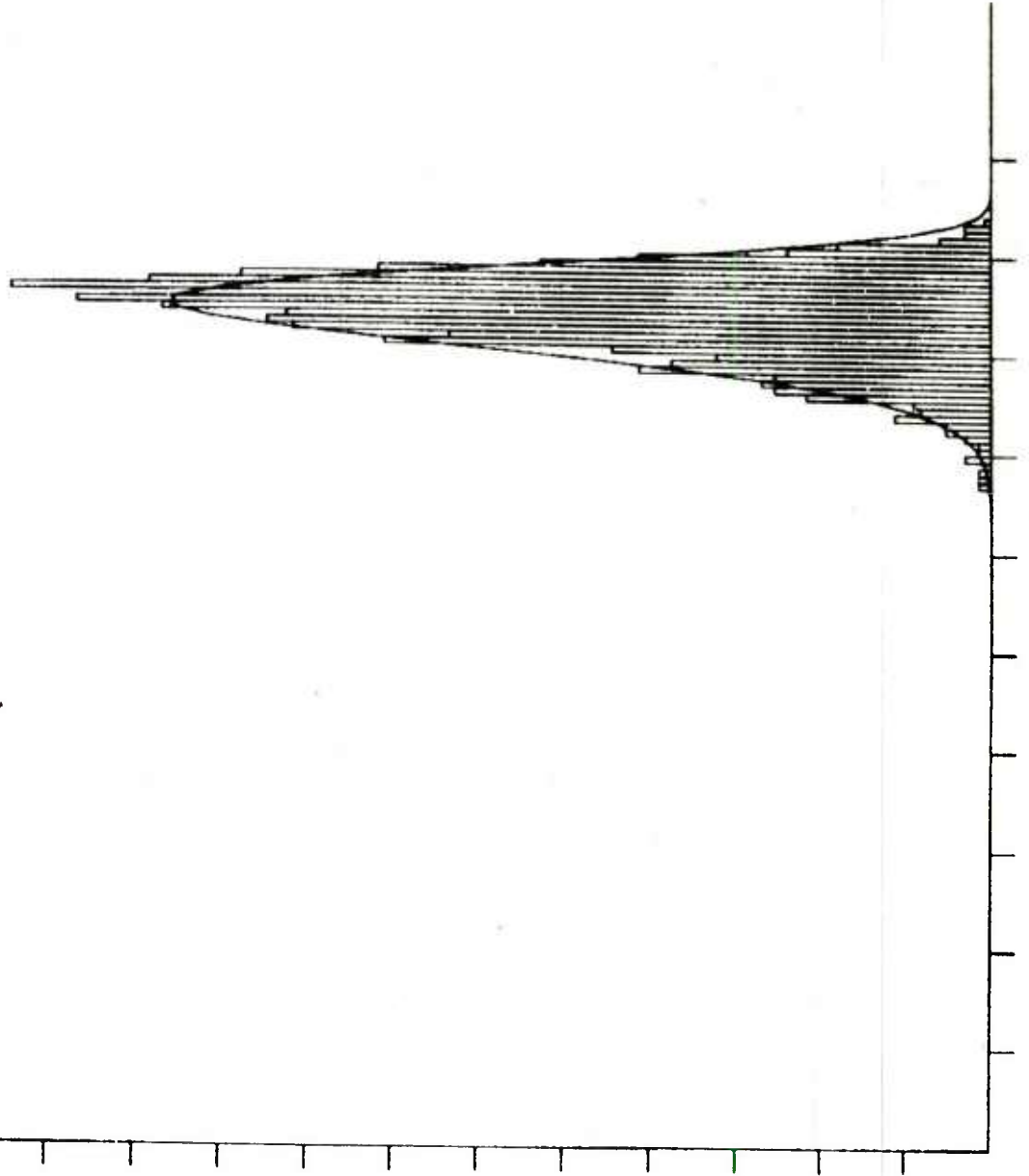
THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .84106
THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .78196
THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .76356

RANDOM END IS 1555639634

PLACE CURSOR ON THIS LINE AND PRESS PROPER KEY FOR DESIRED ACTION

KEY ACTION

- 1 JOB TERMINATION
- 2 NEXT CASE-NEW A AND B
- 3 NEXT CASE-OLD A AND B



b. Use of Four Options in Program Moment

OPTIONS AVAILABLE

- 1 UNIFORM SYSTEM PRIOR
 - 2 USER SPECIFIED EQUAL PRIOR PARAMETERS
 - 3 USER SPECIFIED FIRST AND SECOND SYSTEM MOMENTS
 - 4 USER SPECIFIED UNEQUAL PRIOR PARAMETERS
- OPTION DESIRED - 2

A PARAMETER - 5.0
B PARAMETER - 0.25

THE PRIOR BETA PARAMETERS FOLLOW

THE A IS EQUAL TO 5.00000
THE B IS EQUAL TO .25000

RESULTS USING THE METHOD OF MOMENTS

THE AS IS EQUAL TO 70.50786
THE BS IS EQUAL TO 12.62172
THE MEAN OF RS = .84817
THE MODE OF RS = .85675
THE VAR OF RS = .00153

RANDOM START IS 1234567890

THE NUMBER OF INCREMENTS IS 100.0
NUMBER OF SAMPLES IS 2000.0

CONFIDENCE LIMITS FOR METHOD OF MOMENTS FOLLOW

AS * 70.50786 BS * 12.62172

THE LOWER 50TH PERCENT CONFIDENCE LIMIT * .85095
THE LOWER 90TH PERCENT CONFIDENCE LIMIT * .79618
THE LOWER 95TH PERCENT CONFIDENCE LIMIT * .77925

CONFIDENCE LIMITS FOR MONTE CARLO METHOD FOLLOW

AS * 69.87538 BS * 12.65826
MEAN RS * .84663 VAR RS * .00155
MODE RS * .85524

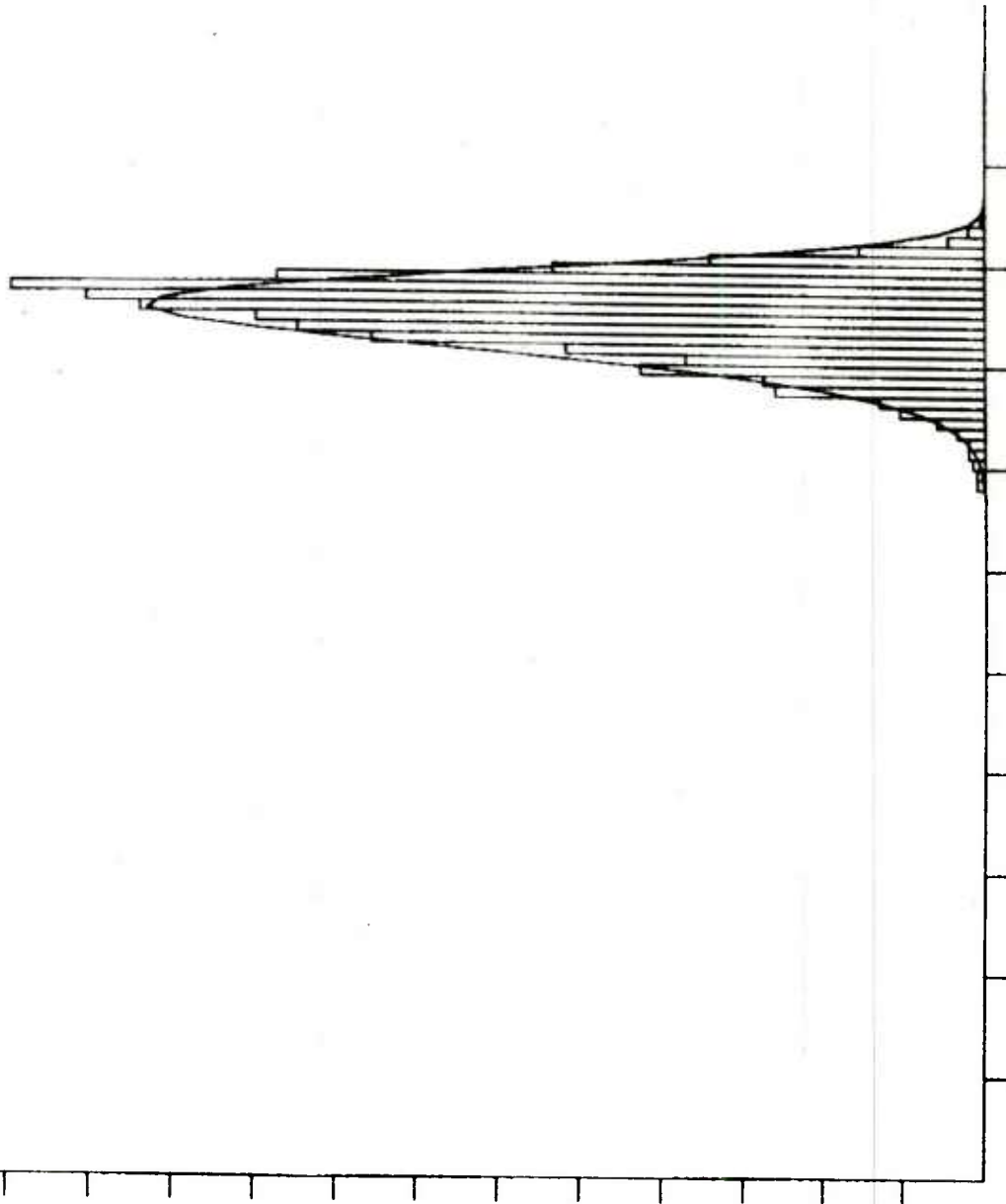
THE LOWER 50TH PERCENT CONFIDENCE LIMIT * .84942
THE LOWER 90TH PERCENT CONFIDENCE LIMIT * .79432
THE LOWER 95TH PERCENT CONFIDENCE LIMIT * .77708

RANDOM END IS 1555639634

PLACE CURSOR ON THIS LINE AND PRESS PROPER KEY FOR DESIRED ACTION

KEY ACTION

- 1 JOB TERMINATION
- 2 NEXT CASE-NEW A AND B
- 3 NEXT CASE-OLD A AND B



OPTIONS AVAILABLE

- 1 UNIFORM SYSTEM PRIOR
- 2 USER SPECIFIED EQUAL PRIOR PARAMETERS
- 3 USER SPECIFIED FIRST AND SECOND SYSTEM MOMENTS
- 4 USER SPECIFIED UNEQUAL PRIOR PARAMETERS

OPTION DESIRED = 3

FIRST MOMENT = 0.82
SECOND MOMENT = 0.82

THE PRIOR BETA PARAMETERS FOLLOW

THE A IS EQUAL TO	7.43650
THE B IS EQUAL TO	.25072

RESULTS USING THE METHOD OF MOMENTS

THE AS IS EQUAL TO	75.09672
THE BS IS EQUAL TO	12.96562
THE MEAN OF RS =	.85277
THE MODE OF RS =	.86097
THE VAR OF RS =	.00141

RANDOM START IS 166622233

THE NUMBER OF INCREMENTS IS 100.0
NUMBER OF SAMPLES IS 1000.0

CONFIDENCE LIMITS FOR METHOD OF MOMENTS FOLLOW

AS = 75.09672 BS = 12.96562

THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .85543
THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .80288
THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .78671

CONFIDENCE LIMITS FOR MONTE CARLO METHOD FOLLOW

AS = 77.64835 BS = 13.38248
MEAN RS = .85299 UAR RS = .00136
MODE RS = .86092

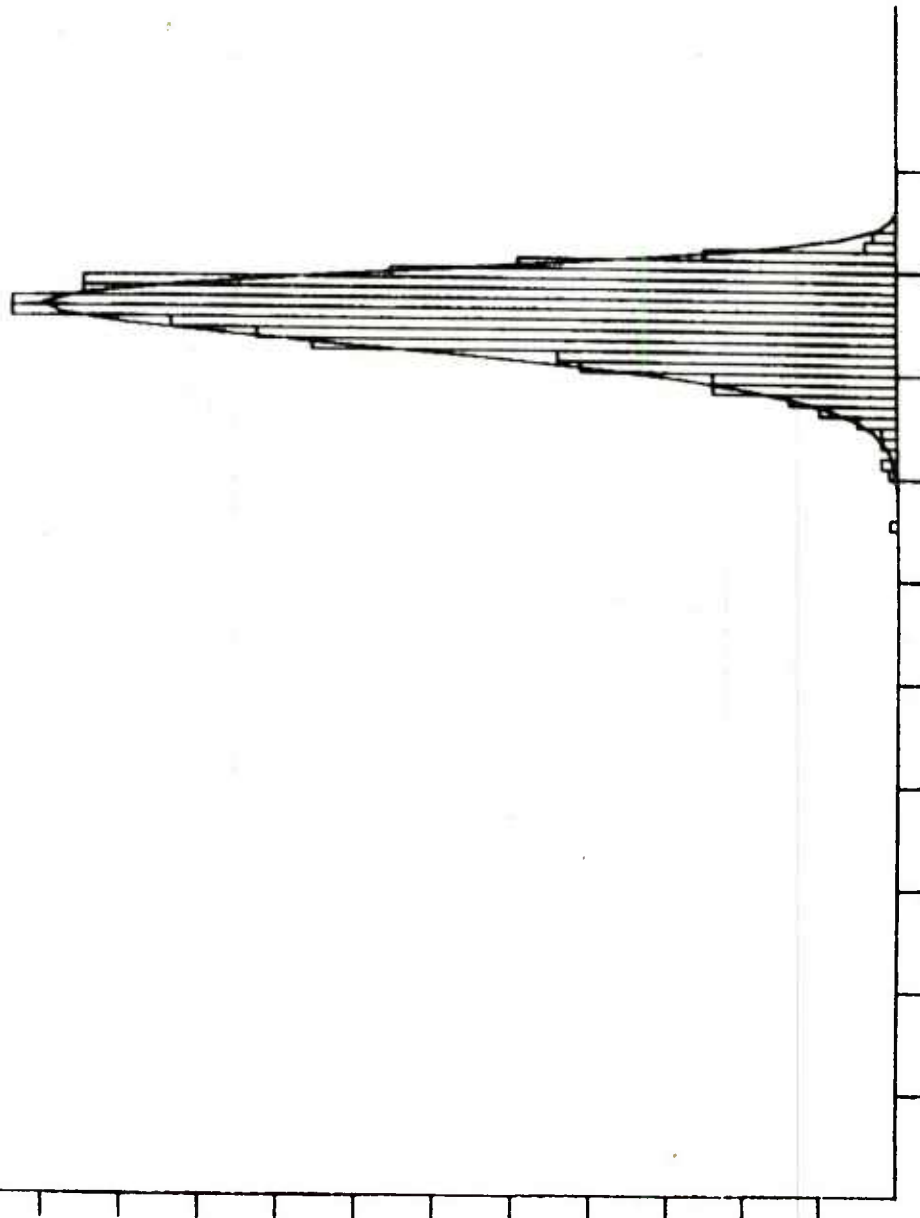
THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .85556
THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .80396
THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .78798

RANDOM END IS 1742760377

PLACE CURSOR ON THIS LINE AND PRESS PROPER KEY FOR DESIRED ACTION

KEY ACTION

- 1 JOB TERMINATION
- 2 NEXT CASE-NEW A AND B
- 3 NEXT CASE-OLD A AND B



OPTIONS AVAILABLE

- 1 UNIFORM SYSTEM PRIOR
- 2 USER SPECIFIED EQUAL PRIOR PARAMETERS
- 3 USER SPECIFIED FIRST AND SECOND SYSTEM MOMENTS
- 4 USER SPECIFIED UNEQUAL PRIOR PARAMETERS

OPTION DESIRED = 4

A(1)	-2.0	B(1)	-2.0
A(2)	-1.0	B(2)	-2.0
A(3)	-2.0	B(3)	-4.0
A(4)	-10.0	B(4)	-10.0
A(5)	-5.0	B(5)	-5.0

THE PRIOR BETA PARAMETERS FOLLOW

A(1) =	2.00000000	B(1) =	2.00000000
A(2) =	1.00000000	B(2) =	2.00000000
A(3) =	2.00000000	B(3) =	4.00000000
A(4) =	10.00000000	B(4) =	10.00000000
A(5) =	5.00000000	B(5) =	5.00000000

RESULTS USING THE METHOD OF MOMENTS

THE AS IS EQUAL TO	51.54638
THE BS IS EQUAL TO	28.67785
THE MEAN OF RS =	.64253
THE MODE OF RS =	.64617
THE VAR OF RS =	.00283

RANDOM START IS 23452345

THE NUMBER OF INCREMENTS IS 100.0
NUMBER OF SAMPLES IS 3000.0

CONFIDENCE LIMITS FOR METHOD OF MOMENTS FOLLOW

AS = 51.54638 BS = 28.67785

THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .64371
THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .57336
THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .55297

CONFIDENCE LIMITS FOR MONTE CARLO METHOD FOLLOW

AS = 49.31461 BS = 27.48389
MEAN RS = .64213 VAR RS = .00295
MODE RS = .64593

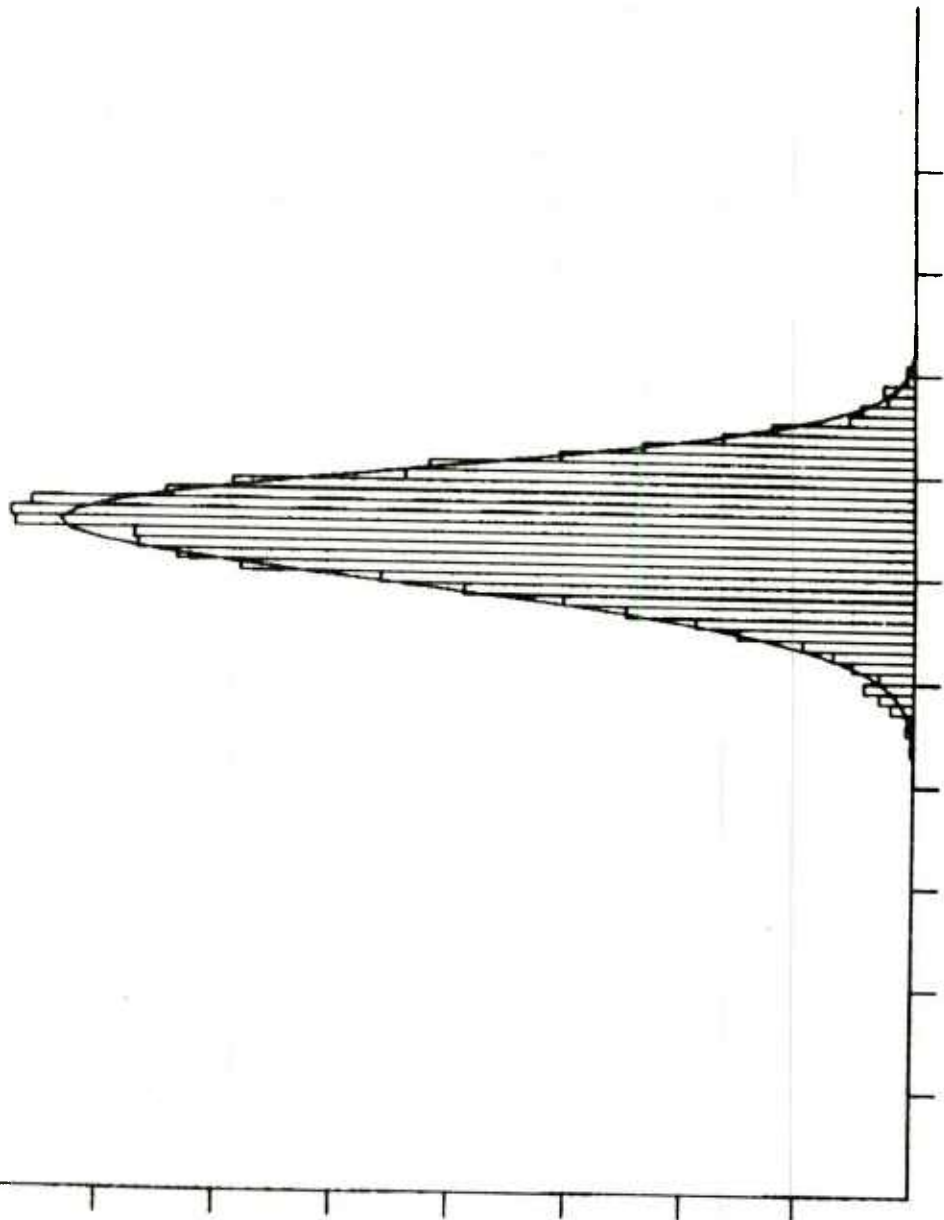
THE LOWER 50TH PERCENT CONFIDENCE LIMIT = .64337
THE LOWER 90TH PERCENT CONFIDENCE LIMIT = .57131
THE LOWER 95TH PERCENT CONFIDENCE LIMIT = .55043

RANDOM END IS 1208952473

PLACE CURSOR ON THIS LINE AND PRESS PROPER KEY FOR DESIRED ACTION

KEY ACTION

- 1 JOB TERMINATION
- 2 NEXT CASE-NEW A AND B
- 3 NEXT CASE-OLD A AND B



APPENDIX A

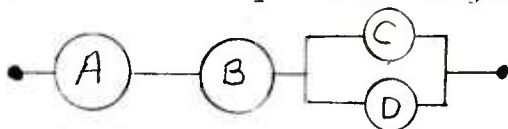
MANUAL COMPUTATION FOR A SIMPLE TEST CASE

MANUAL COMPUTATIONS FOR A SIMPLE TEST CASE

The benefits of BIGRAP can be readily demonstrated by comparing the ease with which BIGRAP is used with the corresponding alternative manual computations. In this appendix the manual computations required to perform just the Method of Moments Bayesian reliability assessment for a simple system configuration will be compared to utilization of BIGRAP. The reader is reminded that BIGRAP also provides the Monte Carlo option for which manual computations are virtually impossible.

Example:

- a. Reliability Block Diagram:



- b. Boolean Expression: $A \cdot B \cdot (C + D)$

(Note: For more complex systems conversion of Boolean expression to algebraic expression is a cumbersome task without the aid of a computer program).

- c. Component Descriptions:

Name	Type, R_i	Successes, s_i	Data Failures, f_i
A	R_1	19	1
B	R_2	17	3
C	R_3		
D	R_3	35	5

- d. Algebraic Reliability Expression for System Reliability, R_s :

$$R_s = 2R_1R_2R_3 - R_1R_2R_3^2$$

- e. Square of Algebraic Expression:

$$R_s^2 = 4R_1^2R_2^2R_3^2 - 4R_1^2R_2^2R_3^3 + R_1^2R_2^2R_3^4$$

- f. First Moment of System Reliability Distribution in Terms of Moments of Component Reliability Distributions:

$$E(R_s) = 2E(R_1)E(R_2)E(R_3) - E(R_1)E(R_2)E(R_3^2)$$

g. Second Moment of System Reliability Distribution:

$$E(R_S^2) = 4E(R_1^2)E(R_2^2)E(R_3^2) - 4E(R_1^2)E(R_2^2)E(R_3^3) \\ + E(R_1^2)E(R_2^2)E(R_3^4)$$

h. Find Component Prior Beta Parameters, a' and b' , Which Yield Uniform Prior Distribution for System Reliability:

$$(1) \text{ Substitute } E(R_i^r) = \prod_{j=1}^r \frac{a'+j-1}{a'+b'+j-1}$$

for each expectation appearing in first and second moment expressions above.

(2) Set resulting first moment expression equal to $1/2$, the value of the first moment of a uniform distribution on the interval $(0,1)$.

(3) Set resulting second moment expression equal to $1/3$, the value of the second moment of the uniform distribution.

(4) Solve the resulting two equations for the two unknowns, a' and b' . For this simple system the two equations to be solved are:

$$E(R_S) = 2 \left(\frac{a'}{a'+b'} \right)^3 - \left(\frac{a'}{a'+b'} \right)^3 \frac{a'+1}{a'+b'+1} = 1/2$$

and

$$E(R_S^2) = 4 \left(\frac{a'}{a'+b'} \right)^3 \left(\frac{a'+1}{a'+b'+1} \right)^3 - 4 \left(\frac{a'}{a'+b'} \right)^3 \left(\frac{a'+1}{a'+b'+1} \right)^3 \frac{a'+2}{a'+b'+2} \\ + \left(\frac{a'}{a'+b'} \right)^3 \left(\frac{a'+1}{a'+b'+1} \right)^3 \left(\frac{a'+2}{a'+b'+2} \right) \left(\frac{a'+3}{a'+b'+3} \right) = \frac{1}{3}$$

To solve these non-linear equations without the use of the computer is a difficult and time consuming task. Iterative procedures such as Newton's Method must be employed.

The solutions for this example are:

$$a' = 1.2382$$

$$b' = 0.3965$$

The values are used as the prior beta parameters for all components and result in an approximately uniform prior distribution for system reliability.

i. Component Posterior Beta Parameters:

$$a_1 = s_1 + a' = 19 + 1.2382 = 20.2382$$

$$b_1 = f_1 + b' = 1 + 0.3965 = 1.3965$$

$$a_2 = s_2 + a' = 17 + 1.2382 = 18.2382$$

$$b_2 = f_2 + b' = 3 + 0.3965 = 3.3965$$

$$a_3 = s_3 + a' = 35 + 1.2382 = 36.2382$$

$$b_3 = f_3 + b' = 5 + 0.3965 = 5.3965$$

j. Component Posterior Moments Needed in First and Second System Moment Expressions:

$$E(R_1) = \frac{a_1}{a_1 + b_1} = \frac{20.2382}{21.6347} = 0.93545$$

$$E(R_1^2) = \frac{a_1}{a_1 + b_1} \cdot \frac{a_1 + 1}{a_1 + b_1 + 1} = 0.87774$$

$$E(R_2) = \frac{a_2}{a_2 + b_2} = 0.84301$$

$$E(R_2^2) = \frac{a_2}{a_2 + b_2} \cdot \frac{a_2 + 1}{a_2 + b_2 + 1} = 0.71651$$

$$E(R_3) = \frac{a_3}{a_3+b_3} = 0.87038$$

$$E(R_3^2) = E(R_3) \cdot \frac{a_3+1}{a_3+b_3+1} = 0.76022$$

$$E(R_3^3) = E(R_3^2) \cdot \frac{a_3+2}{a_3+b_3+2} = 0.66620$$

$$E(R_3^4) = E(R_3^3) \cdot \frac{a_3+3}{a_3+b_3+3} = 0.58565$$

k. First Posterior Moment (Mean) of System Reliability:

Substitute the results in j. in first moment expression provided in f.

$$\begin{aligned} E(R_s) &= 2(0.93545)(0.84301)(0.87038) \\ &\quad - (0.93545)(0.84301)(0.76022) \\ &= 0.77325 \end{aligned}$$

l. Second Posterior Moment of System Reliability:

Substitute the results in j. in the second moment expression provided in g.

$$\begin{aligned} E(R_s^2) &= 4(0.87774)(0.71651)(0.76022) \\ &\quad - 4(0.87774)(0.71651)(0.66620) \\ &\quad + (0.87774)(0.71651)(0.58565) \\ &= 0.604851 \end{aligned}$$

m. Posterior Variance of System Reliability:

$$\begin{aligned} \text{Var}(R_s) &= E(R_s^2) - E^2(R_s) \\ &= 0.604851 - (0.77325)^2 \\ &= 0.006936 \end{aligned}$$

n. System Reliability Posterior Beta Parameters, a_s and b_s :

$$\text{Use } a_s = \frac{E^2(R_s)(1-E(R_s))}{\text{Var}(R_s)} - E(R_s)$$

$$b_s = \frac{E(R_s)(1-E(R_s))^2}{\text{Var}(R_s)} + E(R_s) - 1$$

Hence $a_s = 18.77365$

$b_s = 5.50523$

o. Beta Density Function of Posterior Distribution of System Reliability:

$$f_{\beta}(R_s | a_s, b_s) = f_{\beta}(R_s | 18.77365, 5.50523)$$

$$= \frac{\Gamma(24.27888) R_s^{17.77365} (1-R_s)^{4.50523}}{\Gamma(18.77365) \Gamma(5.50523)}$$

p. Best Estimate of System Reliability:

In this case the posterior distribution has a mode less than 1.0 so that the mode is the best estimate. *

$$\text{Mode of } R_s = \frac{a_s - 1}{a_s + b_s - 2} = 0.79778 \quad (\text{Best Estimate})$$

q. 100(1- γ)% Lower Confidence Limit on System Reliability:

Find R such that

$$\int_0^R f(R_s | a_s, b_s) dR_s = \gamma$$

(Note: Here again manual computations are difficult requiring complex interpolation schemes of tabled incomplete beta values or computer numerical integration algorithms)

*See page 15 for explanation (underlined line)

APPENDIX B

THE SIMPLE TEST CASE AS RUN WITH BIGRAP

CONTROL DATA INTERCOM 4.5
DATE 06/08/78
TIME 09.56.41.

PLEASE LOGIN
LOGIN

ENTER USER NAME-UUUUUXXXXYY

■■■■■■■■■■ ENTER PASSWORD-

06/08/78 LOGGED IN AT 09.57.16.
 WITH USER-ID PG
 EQUIP/PORT 40/006
COMMAND- ETL,300

COMMAND- BEGIN,RELY,BAYE/MARDO,XX,YYY

```

ENTER TOTAL NUMBER OF COMPONENTS- 4
ENTER TOTAL NUMBER OF DIFFERENT COMPONENT TYPES- 3
ENTER NAME OF CASE- MANCHP
ENTER NAME OF EACH COMPONENT-
C( 1 ) -A   C( 2 ) -B   C( 3 ) -C   C( 4 ) -D
FOR EACH COMPONENT ENTER ITS TYPE-
T( 1 ) -R1   T( 2 ) -R2   T( 3 ) -R3   T( 4 ) -R3
FOR EACH TYPE ENTER ITS TOTAL NUMBER OF SUCCESSES
S( 1 ) -19   S( 2 ) -17   S( 3 ) -35
FOR EACH TYPE ENTER ITS TOTAL NUMBER OF FAILURES
F( 1 ) -1    F( 2 ) -3    F( 3 ) -5

ENTER BOOLEAN EXPRESSION
(XXWARNING PLUS SIGN MUST APPEAR INSIDE A SET OF PARENTHESES)
A&B(C+D)
A&B(C+D)

```



```

BOGR  STOP PROGRAM - BOOLEAN ONE (BOLN1)
HAS JUST ENDED
      .518 CP SECONDS EXECUTION TIME
STOP 2
      .097 CP SECONDS EXECUTION TIME
STOP PROGRAM - BOOLEAN THREE (BOLN3
) HAS JUST ENDED
      .184 CP SECONDS EXECUTION TME

```

OPTIONS AVAILABLE

- 1 UNIFORM SYSTEM PRIOR
 - 2 USER SPECIFIED EQUAL PRIOR PARAMETERS
 - 3 USER SPECIFIED FIRST AND SECOND SYSTEM MOMENTS
 - 4 USER SPECIFIED UNEQUAL PRIOR PARAMETERS
- OPTION DESIRED = 1

THE PRIOR BETA PARAMETERS FOLLOW

THE A IS EQUAL TO 1.24080
THE B IS EQUAL TO .39731

RESULTS USING THE METHOD OF MOMENTS

THE AS IS EQUAL TO 18.85078
THE BS IS EQUAL TO 5.52880
THE MEAN OF RS = .77322
THE MODE OF RS = .79764
THE VAR OF RS = .00691

The numerical differences between the results of the computer program and the manual computations can be attributed to the fact that the calculations in the computer program are based on an numerical approximation technique.

■ STOP 4
.602 CP SECONDS EXECUTION TIME
.284 CP SECONDS COMPILATION TIME
PFN IS
TEK30
EOI ENCOUNTERED AFTER COPY OF FILE
0, RECORD 17

RANDOM START IS 1234567890

THE NUMBER OF INCREMENTS IS 120.0
NUMBER OF SAMPLES IS 2000.0

CONFIDENCE LIMITS FOR METHOD OF MOMENTS FOLLOW

AS - 18.85078 BS - 5.52880

THE LOWER 50TH PERCENT CONFIDENCE LIMIT -	.78077
THE LOWER 90TH PERCENT CONFIDENCE LIMIT -	.66129
THE LOWER 95TH PERCENT CONFIDENCE LIMIT -	.62427

CONFIDENCE LIMITS FOR MONTE CARLO METHOD FOLLOW

AS - 18.89870 BS - 5.48380
MEAN RS - .77509 UAR RS - .00687
MODE RS - .79967

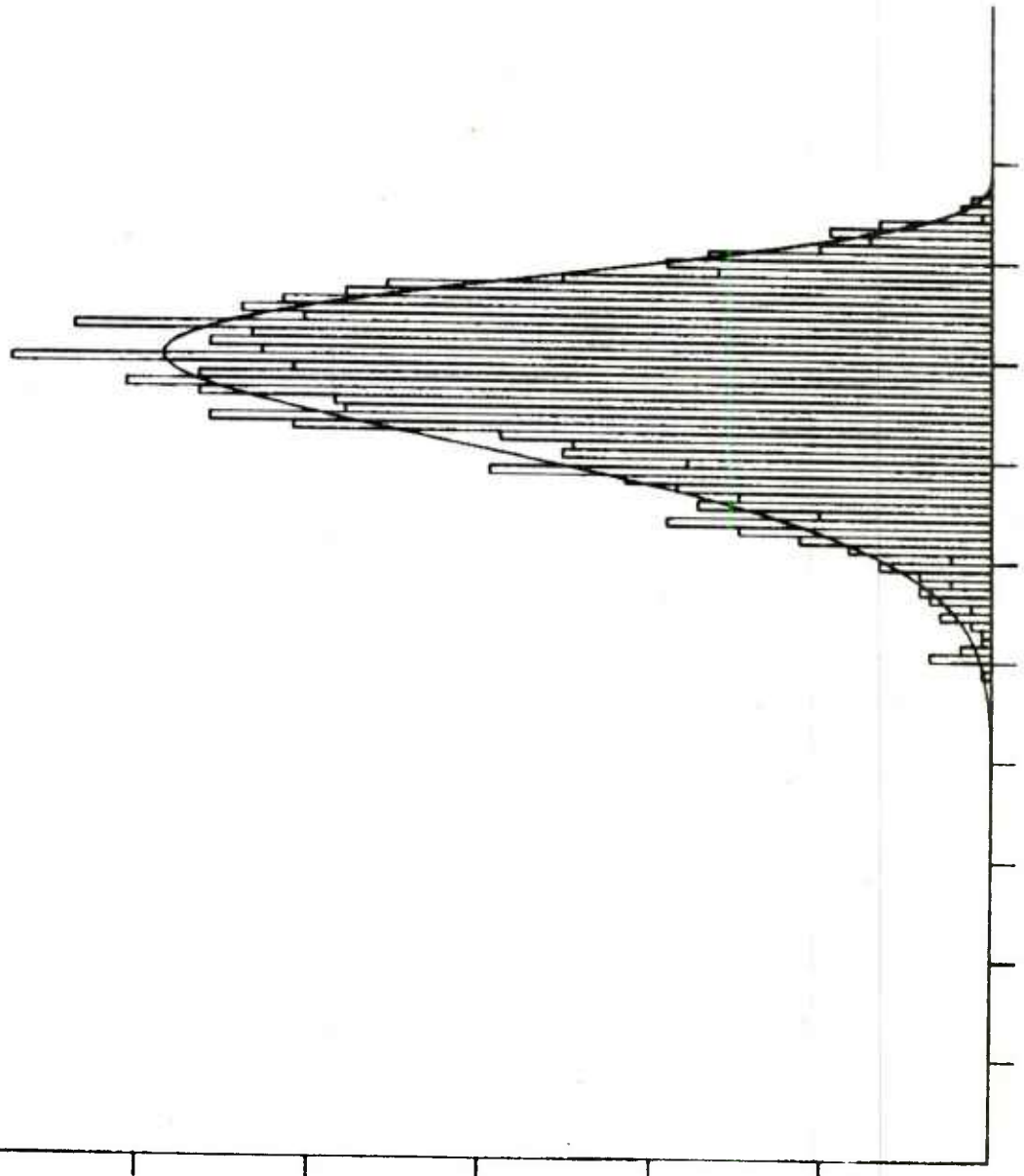
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BIGRAP	Assessment	Component	Statistics
Bayesian	Procedure	Success	Confidence
Interactive	Computer	Failure	Histograms
Graphics	Tektronix 4014	Boolean	Beta
Reliability	ARRADCOM	Logical	Algebraic
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BIGRAP is a package of interactive graphics programs, written for use on the TEKTRONIX 4014 Graphics Terminal connected to the ARRADCOM CDC 6500/6600 computer configuration. This package consists of a set of intricate programs that allows a user to input component success/failure data and a Boolean expression depicting system reliability logic for the purpose of assessing system reliability. The computer converts the logic expression to an			

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20. Abstract (continued)

algebraic expression for the system reliability as a function of the individual component reliabilities. A Bayesian statistical algorithm is then employed to provide the user with point and confidence interval estimates of system reliability. In addition the graphics feature of the package displays histograms and corresponding Beta distributions involved in the analysis. The programs are run sequentially as a cataloged procedure using the BEGIN/REVERT utility. This report is intended to serve as a guide to potential users of BIGRAP for operation and interpretation of results.